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# Comparison of Metaheuristic and Linear Programming Models for the Purpose of Optimising Building Energy Supply Operation Schedule

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## Abstract

*Increasing complexity of building energy systems has led to a wide range of methods to minimise cost of meeting demand for all types of energy. Metaheuristics and mixed integer linear programmes (MILP) are the two most prevalent optimisation methods in the field, with relative advantages which have not previously been compared under common criteria. The principle objective of this paper is to scrutinise these two optimisation methods when applied to a problem of finding the optimal operational schedule of an energy system serving a hotel. 11 technologies are modelled by both methods, but all exhibit nonlinear characteristics which must be linearised for use in MILP. Comparison of the two models results in variation between objective function below 1%, where piecewise linearised MILP gives the most optimal solution. The time to solution varies by orders of magnitude between models: 0.08s for simple linear MILP, 1.64s for piecewise linear MILP and 274s for metaheuristic. System designers, or controllers, must decide between solution time and realistic representation of technologies when choosing an optimisation method, a compromise which may be balanced by piecewise optimisation. Further, Proposed operation schedules vary slightly between methods, allowing some subjectivity in exact operation schedule, without compromising objective function. Metaheuristics favours qualitative subjectivity, while MILP favours quantitative subjectivity.*

**Keywords - Mixed Integer Linear Programming, Epsilon constrained differential evolution, Energy System Optimisation, Metaheuristics**

## 1. Introduction

HVAC systems of large non-domestic buildings often comprise of multiple sub-systems, the operation schedules of which must be optimised for

energy efficiency and cost-effectiveness. The complexity of scheduling optimal operation of such systems increases when multiple combinations of sub-systems can provide the same energy services at any given time. In the optimisation literature, research has demonstrated two different methods to be a viable approach for solving such problems: metaheuristic methods [1]–[4] and the mathematical method of mixed integer linear programming (MILP) [5]–[7]. However, studies have reported challenges associated with each method [2], [6]. Metaheuristic methods can find ‘good-enough’ solutions to highly nonlinear problems in a short time frame by searching a range of possible solution points, but the optimality of the solutions is difficult to guarantee, particularly for more complex systems [2]. Conversely, MILP models require nonlinear behaviours of HVAC systems to be simplified, which can lead to a deterministic and physically incorrect solution. Furthermore, the complex behaviours of certain technologies cannot be modelled. Indeed, commercial establishments were not considered in [5] due to the nonlinear nature of the cooling systems, while second law thermodynamics could not be used in [7] due to the need for temperature tracking. Where simplification occurs, MILP models often utilise rated operational efficiency of technologies used in a system, irrespective of subsequent load rate, e.g. [6].

In energy system modelling, there exists comparisons within the fields of metaheuristics and mathematical modelling, e.g. [8], [9]. However, little literature exists comparing metaheuristic and mathematical methods to each other. Ikeda [2] does so by benchmarking a range of metaheuristic optimisation methods against a dynamic programming (DP) model, but not to the more popular MILP due to the need for nonlinear characteristics. In the power electronics sector, enhanced PSO has been compared to MILP (referred to as the branch & bound method) [10]. However, the PSO inputs were linearised for comparability with the MILP model.

It is evident that comparison has not been previously made between the most prevalent optimisation methods, metaheuristics and MILP, in which the system remains the same but the technology characteristics vary in correspondence with the requirements of the method. As such, the main objective of this paper is to examine the differences between a MILP and metaheuristic model applied to the energy supply optimisation of a non-domestic property, given a common technology portfolio and constraint set. All technologies exhibit some degree of non-linearity in their energy consumption characteristics, which is fully and piecewise linearised for use in two MILP models.

## **2. Case Study and Problem Formulation**

The energy system of a non-domestic building in Tokyo is modelled. A hotel comprising 20,000m<sup>2</sup> total floor space was chosen from the Society of Heating, Air-conditioning and Sanitary Engineers of Japan (SHASE) database



## b. Cooling tower

The TR, AR and HRAR utilise a cooling tower in order to effectively remove heat to the atmosphere. The performance of the cooling tower varies depending on external temperature, which can then be translated to lower average cooling water temperature ( $T_{cw}$ ) and thus a lower average temperature of the respective technology refrigeration cycle, increasing its efficiency. This feedback loop can be modelled in metaheuristic methods as temperature can be tracked and iterations undertaken within the primary optimisation. It is not feasible to emulate this behaviour in MILP; hence, the metaheuristic model must be simplified to not account for  $T_{cw}$  changes. The effect of the omission on the final objective function result will be discussed.

Table 1 Technology characteristics, as provided by SHASE [11].

	Rated capacity	Rated efficiency	Minimum load rate
Natural gas boiler (BB)	750kW	80.0%	0.2
Cogeneration system (CGS)#	352kWe 300kWth	40.5% 34.5%	0.2
Photovoltaic solar panels (PV)	1.2kW/m <sup>2</sup>	13.0%	N/A
Turbo-refrigerator (TR)	2500kW	603%	0.45
Absorption refrigerator (AR)	1500kW	125%	0.25
Heat recovery absorption refrigerator (HRAR)	1000kW	132%	0.30
Air source heat pump (AHP)*	550kW 500kW	358%	0.2
Hot water storage (TES-H)**	1200kWh 360kW	N/A	N/A
Cold water storage (TES-C)**	10000kWh 3000kW	N/A	N/A
Battery storage (B)	500kWh 100kW	N/A	N/A

#CGS produces electricity and heat. \*Two AHPs were used in this study, with differing maximum capacity. \*\*kWh = maximum storage capacity, kW = maximum charge/discharge rate.

## 4. Objective function

The objective function was set to minimise total operation cost as:

$$\sum_p^{24} (\sum_i^{11} (Pe_p * Ec_{i,p}) + \sum_i^{11} (Pg_p * Gc_{i,p}) + Pe_p * Ep_p - Se_p * Es_p). \quad (1)$$

Where  $p$  = hour in day,  $i$  = technology,  $Pe$  = price of purchased electricity,  $Ec$  = technology electricity consumption,  $Pg$  = price of purchased gas,  $Gc$  = technology gas consumption,  $Ep$  = Electricity purchased to meet building electricity demand,  $Se$  = price of sold electricity,  $Es$  = Electricity sold to grid.

## 5. Optimisation methods

### a. Metaheuristic

Metaheuristic methods belong to the research field of artificial intelligence and find the optimum solutions by trial and error. Ordinarily speaking, metaheuristics do not consider constraints, although almost all actual systems have many constraints. Thus, a method that can handle constraints is necessary. Epsilon differential evolution (eDE), developed by Takahama [12] and Mallipeddi [13], is one such method which exhibits efficiency improvements compared to other constraint handling methods.

The algorithm of eDE used in this study is a development on that introduced in [1] and contains seven parts. It is run in MATLAB [14] as follows: 1) scatter 80 individuals into a search domain using random numbers; 2) evaluate an objective function of each individual and set an initial value of epsilon; 3) Consider individuals only from the top 20% of the population, ranked by epsilon level comparison; 4) create a new solution at a constant mutation rate of 0.5, mixing three individuals in accordance with the original algorithm of DE; 5) evaluate the constraint violation that indicates how far the new individual is from a feasible domain; 6) compare the new individual with the old in terms of the objective function or epsilon, the better becomes the old individual in the next iteration; and 7) finally, return to 4), repeating generations up to 5000 times with cross-over rate decreasing exponentially from 0.8 and epsilon having to reach zero at 2000 generations.

### b. Mixed Integer Linear Programming (MILP)

This paper undertakes MILP in the IBM ILOG CPLEX (CPLEX) environment [15], and builds upon the Distributed Energy Network Optimisation (DENO) model developed by Omu & Choudhary, introduced in [6]. CPLEX combines the branch & bound and cutting plane methods [16], in which an optimal *relaxed solution* (RS) is found by relaxing the constraints and pursuing avenues (branches) by re-application of the constraints, in different combinations. When a feasible solution is found along a branch (fitting all constraints) it becomes an *incumbent solution* until another branch is fully explored to find either an *infeasible solution* (constraint violation) or a new incumbent solution. Branches set to have the same result as the incumbent solution are not explored further, and are said to be *fathomed by bound*. Branching continues in this study until the incumbent solution is within 1% of the RS, with CPLEX initially removing some RSs by creating cutting planes, reducing the time needed to find this feasible solution.

The DENO model has been simplified in scale, to the case of one building, but greater complexity has been applied in the representation of the technologies. Piecewise linearisation is undertaken, by use of integrated functions within the modelling environment, to represent variable technology efficiencies.

## 6. Linearisation

The eDE model utilises the fully nonlinear characteristic curves of the technologies in all calculations. Each technology exhibits a degree of nonlinearity, depending on output compared to maximum capacity, which is linearised for use in CPLEX. The simplest linearisation assumes that technology output changes linearly from minimum load-rate to maximum capacity. Piecewise linearisation segments a nonlinear curve into several connected linear curves. Bischi et al showed that piecewise linearisation is a powerful method of representing highly nonlinear technology characteristic curves in energy system modelling. Up to 10 pieces shows minimal increased computation time compared to the complete linear case, but location of piece connections has greater effect [17]. In this study four pieces along the consumption curves are used, set to concentrate on important locations on the curves including minimum load rate; load rate at maximum efficiency; and maximum load rate, with intermediate points placed to ensure reasonable curve following. Figure 2 shows the electricity consumption and COP curves of the TR, from nonlinear curve to simple linearisation. Simple linearisation leads to a large error over the operating range of the TR, constantly overestimating the required electricity input. Piecewise linearisation provides a reasonable fit throughout, maximum absolute error is 5.9kW (1.7%) for CGS gas consumption at 29.5% load rate, while maximum percentage error is 15.9% (0.5kW) for AHP pump consumption at 75.6% load rate. For comparison, the simple linear maximum absolute error is 26.2kW (8.3%) for TR electricity consumption at 72% load rate, while maximum percentage error is 44.0% (4.68kW) for TES-H pump consumption at 46% load rate.

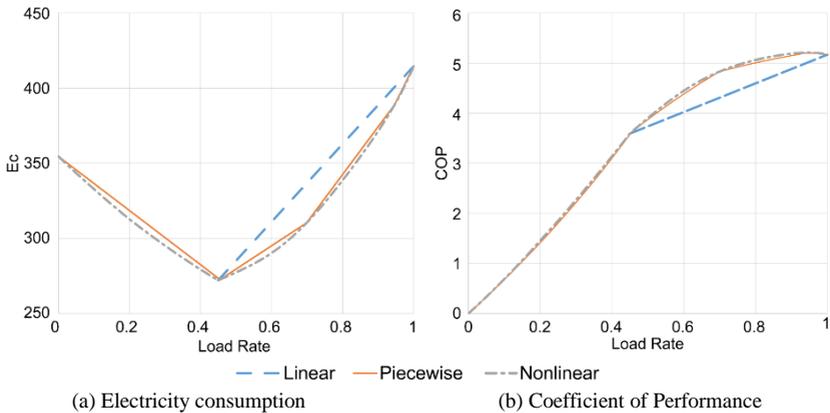


Fig. 2 Comparison of TR characteristic curves by level of linearisation.

## 7. Results

The piecewise linear case offers the best objective function result, albeit only  $0.1\%$  more favourable than the eDE case and  $0.7\%$  more favourable than the simple linear case (table 2).

However, the optimum cases refer to slightly different operation schedules. Figure 3 shows the hot water schedule of both MILP piecewise and eDE, where there is a distinct difference in the utilisation of the TES-H. For instance, the boiler only operates in the first hour in the MILP case, storing heat to use for demand in the subsequent four hours, compared to constant boiler operation and storage charging in the hours 1-4 in the eDE case. Also, the CGS runs for two hours longer in the MILP model, compared to eDE.

Table 2 objective function result for all characteristic curve linearisations.

	MILP simple linear	MILP piecewise linear	eDE
Objective function (JPY)	710,542	705,293	706,013

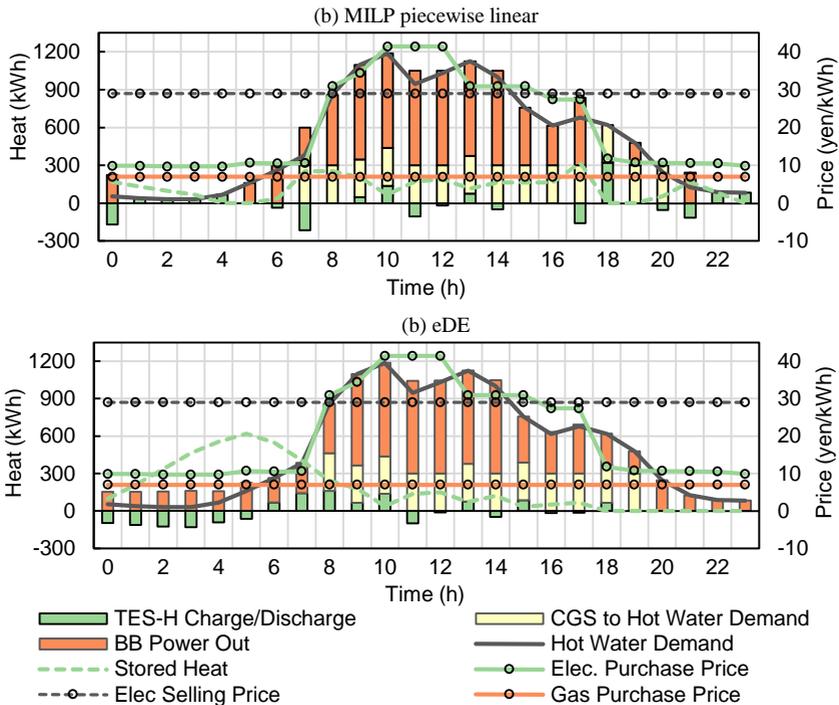


Fig. 3 Comparison of optimum operation schedules for meeting hot water for (a) piecewise linear MILP optimisation and (b) eDE optimisation.

### a. Global Minimum

As the eDE model is unable to find the optimal schedule when initialising decision variables randomly, the operation schedule of the MILP piecewise linear case was used to initialise the eDE decision variables. In doing so, the eDE model begins searching in the solution space in the vicinity of the MILP result. This results in an eDE solution of 705,050 Yen with the same operating schedule as the MILP piecewise linear result (e.g. figure 3a) where the difference of 240 Yen between the two results applied to the same operation schedule is a result of error in piecewise linearisation.

### b. Cooling Water Temperature Variation

When allowing for  $T_{cw}$  to vary in the eDE model ( $\Delta T_{cw}$ ), as discussed previously in section 4, an objective function result of 686,527 Yen is achieved: 3% lower than the MILP piecewise and initial eDE cases whilst following an almost identical schedule to the eDE case. As this variation only affects the cost of cooling, there is likely to be a further reduction in objective function result by initialisation of decision variables, as in 7.a.

### c. Solution Time

Table 3 time to solution for each model run.

	MILP simple linear	MILP piecewise linear	eDE	eDE ( $\Delta T_{cw}$ )
Solution time	0.08s	1.64s	276s	295s

As expected, the increased complexity of technology representation in eDE leads to a greater time to solution, in the order of several minutes (table 3). CPLEX can solve to within 1% of the best bound in under 2 seconds, providing two orders of magnitude improvement on solution time. A further two orders of magnitude improvement are realised in the simplest MILP case. It is evident that the simplification required by MILP allows practically instantaneous results.

## 8. Discussion

This study has shown two primary differences between optimisation methods: the time to solution and operation schedule of hot water supply. Differences in objective function result caused by linearisation is under 1%. Depending on the purpose of an optimised schedule, different solution times are acceptable. An automated system which corrects the schedule in real-time would require an optimisation which can take place within a few seconds, whereas planning for the next-day schedule of meeting building demand requirements could take several minutes. When planning a development, understanding the relevant technologies that should be installed requires that feasible schedules are known. Here, schedule optimisation would be part of a

wider optimisation which includes initial technology purchase and a multi-year outlook and is the least time critical, within reason. In these planning cases operation costs are compared to capital cost of technologies, as such the introduction of  $\Delta T_{cw}$ , which cannot be utilised in the MILP environment, may become pertinent. However, technologies may need more realistic representation in time-constrained circumstances. Knowing that a battery must meet e.g. 30kW or 32kW at a given instant is more important five minutes before the instant than it is during the design phase. This shows a particular mismatch between different requirements, as it is not possible to realise the most accurate technology representation whilst also maintaining a very low time to solution.

We have operation schedules can differ without compromising on objective function (figure 3). Here, there is the possibility for an operator to apply qualitative requirements, or additional quantitative constraints, in order to choose the preferred schedule. As a metaheuristic method, eDE supplies an array of solutions within range of the objective function, so an operator could choose a schedule based on personal preferences (e.g. minimising technology cycling). MILP provides a single optimum, but the speed at which it reaches a solution could allow an operator to add or adapt constraints to create a range of viable scenarios, where weighting on scenario preference will be based on objective function result and subjective preference. The exact method employed will, again, depend on the situation, as availability of variable qualitative/quantitative constraints will differ.

## 9. Conclusions

This paper has compared two popular modelling methods found in the field of energy system modelling: metaheuristics and MILP. By applying linearisation to technology characteristic curves when necessary, the optimal operation schedule of a HVAC system can be found by both a metaheuristic and MILP method. The objective function, to minimise the operating cost of the system over one day, is similar across all methods, although a piecewise linear MILP method is more reliable than metaheuristics for locating a global optimum. Operation schedules given by each method provides small differences that can be exploited by an operator to apply a degree of subjectivity without compromising objective function optimality. However, solution time is larger in the metaheuristic model, by four orders of magnitude compared to a simple linear MILP method. This discrepancy means that for any real time system control, it is unlikely that a complex, realistic representation of technologies is possible. In optimising operation schedules, individuals must decide on criteria such as realistic technology representation, speed, data pre-processing requirements (such as piecewise linearisation), and availability of quantitative/qualitative data before developing a model in an optimisation environment. This paper has gone some way to comparing those criteria.

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